

# Champ moyen quantique et problèmes liés Quantum Mean Field and Related Problems

5-7 Juillet 2017



## LAGA

Laboratoire Analyse Géométrie et Applications

LAGA Université Paris 13, Villetaneuse, France



# BIENVENUE À L'UNIVERSITÉ PARIS 13 CAMPUS DE VILLETANEUSE

UNIVERSITÉ PARIS 13

**LAGA**



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- 2 UFR DSPS - Droit, Sciences Politiques et Sociales.
- 3 UFR SEG - Sciences Économiques et de Gestion
- 4 UFR LLSHS - Lettres Langues Sciences Humaines et des Sociétés
- 5 Institut Galilée
- 6 IUT de Villetaneuse
- 7 DAPS - Département des Activités Physiques et Sportives
- 8 Amphis A B C D (1<sup>er</sup> étage)
- 9 Amphis 1 2 3 4 5 6 7 (1<sup>er</sup> étage)
- 10 Amphis IUT (1<sup>er</sup> étage)

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## Conférenciers

## Speakers

Yvan Castin (LKB, ENS, *Paris*)  
Thomas CHEN (University of Texas, *Austin, USA*)  
Michele CORREGGI (Universita La Sapienza, Roma, *Italy*)  
Marco FALCONI (Universität Zürich, *Switzerland*)  
Stefano LISINI (Univesita degli studii di Pavia, *Italy*)  
Jean NOURRIGAT (Université de Reims, *France*)  
Walter PEDRA (University of Sao Paulo, *Brazil*)  
Peter PICKL (Ludwig Maximilian Universität, Munich, *Germany*)  
Julien SABIN (Université Paris-Sud, *Orsay, France*)  
Benjamin SCHLEIN (Univestität Zürich, *Switzerland*)  
Heinz SIEDENTOP (Ludwig Maximilian Universität, *Munich, Germany*)  
Nikolay TZVETKOV (Université de Cergy, *France*)

## Programme

## Program

Mecredi 5 Wednesday 5	Jeudi 6 Thursday 6	Vendredi 7 Friday 7
11.00 H. SIEDENTOP	10.00 W. PEDRA 11.30 J. NOURRIGAT	10.00 B. SCHLEIN 11.30 T. CHEN
14.00 M. CORREGGI 15.00 J. SABIN 16.30 M. FALCONI	14.30 Y. CASTIN 16.30 P. PICKL	14.30 S. LISINI 16.30 N. TZVETKOV

YVAN CASTIN

LKB, ENS, Paris

*Superfluidity and time coherence of fermions gaz in strong interaction*

As showed Bardeen-Cooper-Schrieffer theory in the fifties, spin  $\frac{1}{2}$  fermions with an attractive interaction and in sufficiently low temperature form Cooper pairs, which can condensate although they are not bosons. The gaz then becomes superfluid and exhibits a coherence length only limited by its size. Since a decade, experiments on trapped cold fermionic atomic gases have motivated works on condensates of fermionic pairs, providing for the first time well isolated quantum systems with a tunable interaction via the Feshbach resonance.

We shall present two conceptual advances allowed by those systems.

The first one is about superfluidity : What is the critical Landau speed of a moving object in a fermionic gaz at zero temperature, velocity below which it fills no friction and moves perpetually in the thermodynamic limit ? We will see that this speed may depend on the mass of the object, contrarily to the case of a weakly interacting bosonic gaz, and therefore exhibits first or second order transition lines in the plan (interaction force, mass). Those predictions have received first confirmations by experiments done Christophe Salomon team in LKB.

The second one is about the coherence time of the pairs condensate at positive temperature, in a well isolated finite system. We shall show that the response depends on the thermodynamical ensemble where the system is prepared. In the canonical ensemble, the coherence time is the one of the phase uncertainty due to the fluctuations of the total energy of different realisations. In the microcanonical ensemble, the coherence time is determined by the phase uncertainty due to the interactions between the bosonic or fermionic quasi-particles, living in the RPA or BCS excitation branches of the fermionic gaz. Those results have not yet been validated experimentally.

## References

- [1] Y. Castin, I. Ferrier-Barbut, C. Salomon, "La vitesse critique de Landau d'une particule dans un superfluide de fermions", *Comptes rendus Physique* 16, 241 (2015) <https://hal.archives-ouvertes.fr/hal-01053706>
- [2] H. Kurkjian, Y. Castin, A. Sinatra, "Brouillage thermique d'un gaz cohérent de fermions", *Comptes Rendus Physique* 17, 789 (2016) <https://hal.archives-ouvertes.fr/hal-01118346>

THOMAS CHEN

Univesity of Texas, Austin

*Fluctuation dynamics around Bose-Einstein condensates and HFB equations*

In this talk, we discuss an extension to the Hartree equation, which describes thermal fluctuations around the Bose-Einstein condensate. Using quasifree reduction, we derive the Hartree-Fock-Bogoliubov (HFB) equations, and discuss the well-posedness of the corresponding Cauchy problem. In particular, the emergence of Bose-Einstein condensates at positive temperature via a self-consistent Gibbs state is addressed.

This is based on joint work with V. Bach, S. Breteaux, J. Froehlich, and I.M. Sigal.

## MICHELE CORREGGI

Universita la Sapienza, Roma

### *Effective Potential Generated by Field Interaction in the Quasi-Classical Limit*

We study the quasi-classical limit of a quantum system composed of finitely many non-relativistic particles coupled to a quantized field in Nelson-type models. We prove that, as the field becomes classical and the corresponding degrees of freedom are traced out, the effective Hamiltonian of the particles converges in resolvent sense to a self-adjoint Schroedinger operator with an additional potential, depending on the state of the field. Moreover, we explicitly derive the expression of such a potential for a large class of field states and show that, for certain special sequences of states, the effective potential is trapping. In addition, we prove convergence of the ground state energy of the full system to a suitable effective variational problem involving the classical state of the field.

## MARCO FALCONI

Universität Zürich

### *Cylindrical Wigner Measures in Bosonic systems*

The semiclassical structure associated to infinite dimensional Heisenberg groups plays an important role in the effective analysis of bosonic quantum field theories. The Wigner measures associated to regular quantum states reveal some interesting features that are hidden for finite dimensional Heisenberg groups, as well as the prominent role played by the projective structure of finite dimensional quotients of the classical phase space. In this talk I will introduce the cylindrical Wigner measures, and discuss sufficient conditions on the quantum states that ensure their concentration as Radon measures in some given "interesting" space. Finally, I will discuss some applications to variational semiclassical problems.

## STEFANO LISINI

Universita degli studii di Pavia

### *Mean-field approximation of controlled Vlasov-type dynamics*

In this talk I will describe a mean-field approximation of a controlled Vlasov-type dynamics. I will describe a suitable discrete Gamma-limit approximation in order to show the commutation of the mean-field limit with the optimization. Techniques of optimal transportation of measures are used.

Joint work with M. Fornasier, C. Orrieri and G. Savaré.

## JEAN NOURRIGAT

Université de Reims

### *Quantum and Classical Evolutions in Nuclear Magnetic Resonances*

In this joint work with Laurent Amour and Lisette Jagger, we derive the classical Maxwell-Bloch model for nuclear magnetic resonances from another model introduced within quantum field theory. While the Bloch model corresponds to the main term in the semiclassical asymptotics, the full expansion of correcting terms is also provided and the photon number is analyzed.

Joint work with Laurent Amour and Lisette Jagger.

## WALTER PEDRA

University of Sao Paulo

*Thermodynamical Stability and Dynamics of Lattice Fermions with Long-Range Interactions*

TBA

## PETER PICKL

Ludwig Maximilian Universität, München

*Dynamics of Sound Waves in an Interacting Bose Gas*

We consider a non-relativistic quantum gas of  $N$  bosonic atoms confined to a box of volume  $\Lambda$  in physical space. The atoms interact with each other through a pair potential whose strength is inversely proportional to the density,  $\rho = \frac{N}{\Lambda}$ , of the gas. We study the time evolution of coherent excitations above the ground state of the gas in a regime of large volume  $\Lambda$  and small ratio  $\frac{\Lambda}{\rho}$ . The initial state of the gas is assumed to be close to a product state of one-particle wave functions that are approximately constant throughout the box. The initial one-particle wave function of an excitation is assumed to have a compact support independent of  $\Lambda$ . We derive an effective non-linear equation for the time evolution of the one-particle wave function of an excitation and establish an explicit error bound tracking the accuracy of the effective non-linear dynamics in terms of the ratio  $\frac{\Lambda}{\rho}$ . We conclude with a discussion of the dispersion law of low-energy excitations, recovering Bogolyubov's well-known formula for the speed of sound in the gas, and a dynamical instability for attractive two-body potentials.

## JULIEN SABIN

Université de Paris Sud, Orsay

*Spectral cluster bounds for density matrices*

Sogge's  $L^p$  bounds are a way to measure the concentration of eigenfunctions of the Laplace-Beltrami operator on compact Riemannian manifolds associated to large eigenvalues. We generalize these bounds to systems of orthonormal functions (or more generally, to density matrices), building a bridge between Sogge's result about concentration and the Weyl law, which in some sense is a manifestation of non-concentration. The optimality of these new bounds is also discussed. These spectral cluster bounds follow from Schatten-type bounds on oscillatory integral operators.

Joint work with Rupert Frank (Caltech).

## BENJAMIN SCHLEIN

Universität Zürich

*Dynamical and spectral properties of Bose gases*

We consider systems of  $N$  bosons interacting through a repulsive potential  $N^{3\beta-1}V(N^\beta x)$  for some  $\beta \in (0; 1]$ . For  $\beta = 1$ , we recover the well-known Gross-Pitaevskii regime. We present new techniques that allow us to prove the convergence towards the time-dependent Gross-Pitaevskii equation with optimal rate. Furthermore, we explain how, for small potentials, this approach can be used to show complete Bose-Einstein condensation (with a uniform bound on the number of excitations) for the ground state and, more generally, for states with small excitation energy. For  $\beta < 1$ , the same method can be used to establish the validity of Bogoliubov theory for the low-lying excitation spectrum.

This talk is based on joint works with C. Boccato, C. Brennecke and S. Cenatiempo.

## HEINZ SIEDENTOP

Ludwig Maximilian Universität, München

### *Effective mean field of atoms close to the nucleus*

I will present an overview on the behavior of the electron density of atoms close to the nucleus which goes in the literature under the name of "Strong Scott Conjecture".

## NIKOLAY TZVETKOV

Université de Cergy

### *On the transport of Gaussian measures under the flow of Hamiltonian partial differential equations*

We will present recent results concerning the description of the transport of natural Gaussian measures under Hamiltonian dynamics in infinite dimensional spaces. The methods consist in combination of probabilistic tools as the ones used in QFT together with PDE tools such as energy estimates and functional spaces capturing the dispersive effects.

## Organisateurs --- Organizing Committee

Zied AMMARI (*Université de Rennes 1, France*)

Sébastien BRETEAUX (*BCAM, Basque Country, Spain*)

Quentin LIARD (*Université de Paris 13, France*)

Francis NIER (*Université de Paris 13, France*)

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