Université Sorbonne Paris Nord, 2022 – 23
Licence de mathématiques 2^{nd} year
Probabilities

Exercise sheet n°4 : Introduction to continuous random variables

Exercise 1

1. Find the cumulative distribution function of the (discrete) uniform distribution on $\{1, \ldots, n\}$. Plot the function for $n = 5$.
2. Let X be a uniform distribution on $[0; 1]$. Find its cumulative distribution function.

Exercise 2 Let X be the exponential distribution of parameter 1.

1. Find the cumulative distribution function of X using its density.
2. Find the cumulative distribution function of X^2.
3. What is its density ?
4. Find the density of X^n, for all $n \in \mathbb{N}^*$.

Exercise 3 Let U be the uniform distribution on $[0, 1]$.

1. Let $n \in \mathbb{N}^*$. Show that $[nU]$ is a uniform distribution on $\{0, 1, \ldots, n-1\}$ (for all $x \in \mathbb{R}$, $[x]$ is the floor function of x).
2. Let $a > 0$. Find the distribution $-\frac{1}{a}\ln(U)$.

Exercise 4 Let $X \sim \mathcal{N}(\mu, \sigma^2)$, for $\mu \in \mathbb{R}$ and $\sigma^2 > 0$.

1. Express the cumulative distribution function of $\frac{X-\mu}{\sqrt{\sigma^2}}$ as an integral.
2. Using a change of variables under the integral sign, show that $\frac{X-\mu}{\sqrt{\sigma^2}} \sim \mathcal{N}(0, 1)$.

Exercise 5 Let X be a point chosen uniformly from $[0, 2]$. What is the probability that the equilateral triangle whose sides have length X has a surface greater than 1 ?

Exercise 6 Let $n \in \mathbb{N}^*$ and G_n be a geometric distribution of parameter $p = 1/n$.

1. Compute $\mathbb{P}(G_n > k)$, $\forall k \in \mathbb{N}$.
2. For all positive real number x and for all integer n, we denote by $[nx]$ the floor function of nx. Compute
$$\lim_{n\to\infty} \mathbb{P}(G_n > [nx]).$$
We denote by $\ell(x)$ the above limit.
3. The function $F : x \in \mathbb{R}_+ \mapsto 1 - \ell(x)$ (and $F : x \in \mathbb{R}_-^* \mapsto 0$) is the cumulative distribution function of a "classical" random variable. Which one is it ? Justify your response.