Miscellaneous Integrals (Answers)

Instructions: For each of the following, you should be able to perform each of the integrations and differentiate the result to get back to the original integrand **without any errors**.

1. $\displaystyle\int \frac{1}{(1+x)\sqrt{x}}\,dx = 2\tan^{-1}(\sqrt{x}) + C$
2. $\displaystyle\int \frac{\sec^2 x}{1+\tan x}\,dx = -\ln\left(\tan\left(\frac{x}{2}\right)-1\right) - \ln\left(1+\tan\left(\frac{x}{2}\right)\right) + \ln\left(\tan^2\left(\frac{x}{2}\right) - 1 - 2\tan\left(\frac{x}{2}\right)\right) + C$
3. $\displaystyle\int \sin x \sec x\,dx = -\ln(\cos x) + C$
4. $\displaystyle\int \frac{\csc x \cot x}{1+\csc^2 x}\,dx = -\tan^{-1}(cscx) + C$
5. $\displaystyle\int \frac{\tan x}{\cos^2 x}\,dx = \frac{1}{2}sec^2x + C$
6. $\displaystyle\int \csc^4 x\,dx = -\frac{cosx}{3\sin^3 x} - \frac{2\cos x}{3\sin x} + C$
7. $\displaystyle\int x\tan^2 x\,dx = -\frac{x^2}{2} + x\tan x - \frac{1}{2}\ln(1+\tan^2 x) + C$
8. $\displaystyle\int x^2\cos^2 x\,dx = \frac{x^2}{2}\cos x\sin x + \frac{x^3}{6} + \frac{1}{2}x\cos^2 x - \frac{1}{4}\cos x\sin x - \frac{x}{4} + C$
9. $\displaystyle\int x^5\sqrt{2-x^3}\,dx = -\frac{2}{45}(2-x^3)(3x^3+4)\sqrt{2-x^3} + C$
10. $\displaystyle\int \frac{1}{\sqrt{x^2+4}}\,dx = \sinh^{-1}\left(\frac{x}{2}\right) + C$
11. $\displaystyle\int \frac{x^2}{\sqrt{25+x^2}}\,dx = \frac{x}{2}\sqrt{25+x^2} - \frac{25}{2}\sinh^{-1}\left(\frac{x}{5}\right) + C$
12. $\displaystyle\int (\cos x)\sqrt{4-\sin^2 x}\,dx = \frac{1}{2}\sin x\sqrt{4-\sin^2 x} + 2\sin^{-1}\left(\frac{1}{2}\sin x\right) + C$
13. $\displaystyle\int \frac{1}{x^2-x+1}\,dx = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$
14. $\displaystyle\int \sqrt{x^2+x+1}\,dx = \frac{2x+1}{4}\sqrt{x^2+x+1} + \frac{3}{8}\sinh^{-1}\left(\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)\right) + C$
15. $\displaystyle\int \frac{5x+31}{3x^2-4x+11}\,dx = \frac{5}{6}\ln(3x^2-4x+11) + \frac{103}{87}\sqrt{29}\tan^{-1}\left(\frac{\sqrt{29}}{58}(6x-4)\right) + C$
16. $\displaystyle\int \frac{x^4+1}{x^2+1}\,dx = \frac{x^3}{3} - x + 2\tan^{-1}x + C$
17. $\displaystyle\int \sqrt{x^4+x^7}\,dx = \frac{2\sqrt{x^4+x^7}(1+x^3)}{9x^2} + C$
18. $\displaystyle\int \frac{\sqrt{x}}{1+x}\,dx = 2\sqrt{x} - 2\tan^{-1}(\sqrt{x}) + C$
19. $\displaystyle\int \frac{\cos x}{\sqrt{4-\sin^2 x}}\,dx = \sin^{-1}\left(\frac{\sin x}{2}\right) + C$

20. $\displaystyle\int \frac{\cos 2x}{\cos x}\,dx = 2\sin x - \ln(\sec x + \tan x) + C$

21. $\displaystyle\int \frac{\tan x}{\ln(\cos x)}\,dx = -\ln(\ln(\cos x)) + C$

22. $\displaystyle\int \frac{x^7}{\sqrt{1-x^4}}\,dx = -\frac{x^4+2}{6}\sqrt{1-x^4} + C$

23. $\displaystyle\int \ln(1+x)\,dx = (x+1)(\ln(x+1)-1) + C$

24. $\displaystyle\int x\sec^{-1} x\,dx = \frac{x^2}{2}\sec^{-1} x - \frac{1}{2}\sqrt{x^2-1} + C$

25. $\displaystyle\int \sqrt{x^2+9}\,dx = \frac{x}{2}\sqrt{x^2+9} + \frac{9}{2}\sinh^{-1}\left(\frac{x}{3}\right) + C$

26. $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx = -\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right) + C$

27. $\displaystyle\int \sqrt{2x-x^2}\,dx = \frac{1}{2}(x-1)\sqrt{2x-x^2} + \frac{1}{2}\sin^{-1}(x-1) + C$

28. $\displaystyle\int \frac{4x-2}{x^3-x}\,dx = 2\ln x + \ln(x-1) - 3\ln(x+1) + C$

29. $\displaystyle\int \frac{x^4}{x^2-2}\,dx = \frac{x^3}{3} + 2x - 2\sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$

30. $\displaystyle\int \frac{\sec x\tan x}{\sec x + \sec^2 x}\,dx = \ln(\sec x) + \ln(x-1) - 3\ln(x+1) + C$

31. $\displaystyle\int \frac{x}{(x^2+2x+2)^2}\,dx = -\frac{x+2}{2(x^2+2x+2} - \frac{1}{2}\tan^{-1}(x+1) + C$

32. $\displaystyle\int \frac{x^{1/3}}{x^{1/2}+x^{1/4}}\,dx = \frac{6}{5}x^{5/6} - \frac{12}{7}x^{7/12} + 3x^{1/3} - 12x^{1/12} + 4\ln(1+x^{1/12}) - 2\ln(x^{1/6} - x^{1/12} + 1) + 4\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}(2x^{1/12}-1)\right) + C$

33. $\displaystyle\int \frac{1}{1+\cos 2x}\,dx = \frac{1}{2}\tan x + C$

34. $\displaystyle\int \frac{\sec x}{\tan x}\,dx = \ln(\csc x - \cot x) + C$

35. $\displaystyle\int \sec^3 x\tan^3 x\,dx = \frac{3-5\cos^2 x}{15\cos^5 x} + C$

36. $\displaystyle\int x^2\tan^{-1} x\,dx = \frac{x^3}{3}\tan^{-1} x - \frac{x^2}{6} + \frac{1}{6}\ln(x^2+1) + C$

37. $\displaystyle\int x(\ln x)^3\,dx = \frac{x^2}{2}\ln^3 x - \frac{3}{4}x^2\ln^2(x) + \frac{3}{4}x^2\ln x - \frac{3x^2}{8} + C$

38. $\displaystyle\int \frac{1}{x\sqrt{1+x^2}}\,dx = -\tanh^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right) + C$

39. $\displaystyle\int e^x\sqrt{1+e^{2x}}\,dx = \frac{e^x}{2}\sqrt{1+e^{2x}} + \frac{1}{2}\sinh^{-1}(e^x) + C$

40. $\displaystyle\int \frac{x}{\sqrt{4x-x^2}}\,dx = -\sqrt{4x-x^2} + 2\sin^{-1}\left(-1+\frac{x}{2}\right) + C$

41. $\displaystyle\int \frac{1}{x^3\sqrt{x^2-9}}\,dx = \frac{1}{18}\frac{\sqrt{x^2-9}}{x^2} - \frac{1}{54}\arctan\left(\frac{3}{\sqrt{x^2-9}}\right) + C$

42. $\displaystyle\int \frac{x}{(7x+1)^{17}}\,dx = \frac{1}{784}\,(7\,x+1)^{-16} - \frac{1}{735}\,(7\,x+1)^{-15} + C$

43. $\displaystyle\int \frac{4x^2+x+1}{4x^3+x}\,dx = \ln x + \frac{1}{2}\tan^{-1}(2x) + C$

44. $\displaystyle\int \frac{4x^3-x+1}{x^3+1}\,dx = 4x - \frac{2}{3}\ln(x+1) + \frac{1}{2}\ln(x^2-x+1) - \frac{4}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$

45. $\displaystyle\int \tan^2 x\sec x\,dx = \frac{1}{2}\sin x\tan^2 x + \frac{1}{2}\sin x - \frac{1}{2}\ln(\sec x+\tan x) + C$

46. $\displaystyle\int \frac{x^2+2x+2}{(x+1)^3}\,dx = \frac{-1}{2(x+1)^2} + \ln(x+1) + C$

47. $\displaystyle\int \frac{x^4+2x+2}{x^5+x^4}\,dx = -\frac{2}{3x^3} + \ln(x+1) + C$

48. $\displaystyle\int \frac{8x^2-4x+7}{(x^2+1)(4x+1)}\,dx = 2\ln(4x+1) - \tan^{-1}x + C$

49. $\displaystyle\int \frac{3x^5-x^4+2x^3-12x^2-2x+1}{(x^3-1)^2}\,dx = \frac{1}{x-1} + \ln(x-1) + \ln(x^2+x+1) - \frac{2}{x^2+x+1} + C$

50. $\displaystyle\int \frac{x}{x^4+4x^2+8}\,dx = \frac{1}{4}\tan^{-1}\left(\frac{x^2}{2}+1\right) + C$

51. $\displaystyle\int (\ln x)^6\,dx = x(\ln x)^6 - 6x(\ln x)^5 + 30x(\ln x)^4 - 120x(\ln x)^3 + 360x(\ln x)^2 - 720x\ln x + 720x + C$

52. $\displaystyle\int \frac{(1+x^{2/3})^{3/2}}{x^{1/3}}\,dx = \frac{3}{5}\left(1+x^{2/3}\right)^{5/2} + C$

53. $\displaystyle\int \frac{(\arcsin x)^2}{\sqrt{1-x^2}}\,dx = \frac{1}{3}(\sin^{-1}x)^3 + C$

54. $\displaystyle\int \frac{1}{x^{3/2}(1+x^{1/3})}\,dx = \frac{-2}{\sqrt{x}} + \frac{6}{x^{1/6}} + 6\tan^{-1}(x^{1/6}) + C$

55. $\displaystyle\int \tan^3 x\,dx = \frac{1}{2}\tan^2 x - \frac{1}{2}\ln(1+\tan^2 x) + C$

56. $\displaystyle\int \sin^2 x\cos^4 x\,dx = -\frac{1}{6}\sin x\cos^5 x + \frac{1}{24}\cos^3 x\sin x + \frac{1}{16}\cos x\sin x + \frac{1}{16}x + C$

57. $\displaystyle\int \frac{xe^{x^2}}{1+e^{2x^2}}\,dx = \frac{1}{2}\tan^{-1}(e^{x^2}) + C$

58. $\displaystyle\int \frac{\cos^3 x}{\sqrt{\sin x}}\,dx = -\frac{2}{5}(\sin x)^{5/2} + 2\sqrt{\sin x} + C$

59. $\displaystyle\int x^3 e^{-x^2}\,dx = -\frac{1}{2}e^{-x^2}(x^2+1) + C$

60. $\displaystyle\int \sin\sqrt{x}\,dx = 2\sin(\sqrt{x}) - 2\sqrt{x}\cos(\sqrt{x}) + C$

61. $\displaystyle\int \frac{\arcsin x}{x^2}\,dx = -\frac{\sin^{-1} x}{x} - \tanh^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) + C$

62. $\displaystyle\int x^2\sqrt{1-x^2}\,dx = -\frac{x}{4}(1-x^2)^{3/2} + \frac{x}{8}\sqrt{1-x^2} + \frac{1}{8}\sin^{-1} x + C$

63. $\displaystyle\int \sqrt{x^2-9}\,dx = \frac{x}{2}\sqrt{x^2-9} - \frac{9}{2}\ln\left(x+\sqrt{x^2-9}\right) + C$

64. $\displaystyle\int x\sqrt{2x-x^2}\,dx = -\frac{1}{2}(2x-x^2)^{3/2} + \frac{x-1}{2}\sqrt{2x-x^2} + \frac{1}{2}\sin^{-1}(x-1) + C$

65. $\displaystyle\int \frac{x-2}{4x^2+4x+1}\,dx = \frac{5}{4(1+2x)} + \frac{1}{4}\ln(1+2x) + C$

66. $\displaystyle\int \frac{2x^2-5x-1}{x^3-2x^2-x+2}\,dx = 2\ln(x-1) - \ln(x-2) + \ln(x+1) + C$

67. $\displaystyle\int \frac{e^{2x}}{e^{2x}-1}\,dx = \frac{1}{2}\ln(e^{2x}-1) + C$

68. $\displaystyle\int \frac{\cos x}{\sin^2 x - 3\sin x + 2}\,dx = -\ln(-1+\sin x) + \ln(-2+\sin x) + C$

69. $\displaystyle\int \frac{2x^3+3x^2+4}{(x+1)^4}\,dx = -\frac{5}{3(x+1)^3} + \frac{3}{x+1} + 2\ln(x+1) + C$

70. $\displaystyle\int \frac{\sec^2 x}{\tan^2 x + 2\tan x + 2}\,dx = \tan^{-1}(1+\tan x) + C$

71. $\displaystyle\int \frac{x^3+x^2+2x+1}{x^4+2x^2+1}\,dx = -\frac{1}{2(x^2+1)} + \frac{1}{2}\ln(x^2+1) + \tan^{-1} x + C$

72. $\displaystyle\int \sin x\cos 3x\,dx = -\frac{1}{8}\cos(4x) + \frac{1}{4}\cos(2x) + C$

73. $\displaystyle\int x^5\sqrt{x^3-1}\,dx = \frac{2}{45}(3x^6-x^3-2)\sqrt{x^3-1} + C$

74. $\displaystyle\int \ln(x^2+2x)\,dx = x\ln(x^2+2x) - 2x + 2\ln(x+2) + C$

75. $\displaystyle\int \frac{\sqrt{1+\sin x}}{\sec x}\,dx = \frac{2}{3}(1+\sin x)^{3/2} + C$

76. $\displaystyle\int \frac{1}{x^{2/3}(1+x^{2/3})}\,dx = 3\tan^{-1}(x^{1/3}) + C$

77. $\displaystyle\int \frac{\sin x}{\sin 2x}\,dx = \frac{1}{2}\ln(\sec x + \tan x) + C$

78. $\displaystyle\int \sqrt{1+\cos x}\,dx = 2\sqrt{2}\sin\left(\frac{x}{2}\right) + C$

79. $\displaystyle\int \sqrt{1+\sin x}\,dx = \frac{2(-1+\sin x)\sqrt{1+\sin x}}{\cos x} + C$

80. $\displaystyle\int \frac{\sec^2 x}{1-\tan^2 x}\,dx = -\frac{1}{2}\ln\left(\tan^2\left(\frac{x}{2}\right) + 2\tan\left(\frac{x}{2}\right) - 1\right) + \frac{1}{2}\ln\left(\tan^2\left(\frac{x}{2}\right) - 2\tan\left(\frac{x}{2}\right) - 1\right) + C$

81. $\displaystyle\int \ln(x^2+x+1)\,dx = x\ln(x^2+x+1) - 2x + \frac{1}{2}\ln(x^2+x+1) + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) + C$

82. $\displaystyle\int e^x \sin^{-1}(e^x)\,dx = e^x \sin^{-1}(e^x) + \sqrt{1-e^{2x}} + C$

83. $\displaystyle\int \frac{\arctan x}{x^2}\,dx = -\frac{\tan^{-1} x}{x} + \ln x - \frac{1}{2}\ln(x^2+1) + C$

84. $\displaystyle\int \frac{x^2}{\sqrt{x^2-25}}\,dx = \frac{x}{2}\sqrt{x^2-25} + \frac{25}{2}\ln\left(x + \sqrt{x^2-25}\right) + C$

85. $\displaystyle\int \frac{x^3}{(x^2+1)^2}\,dx = \frac{1}{2(x^2+1)} + \frac{1}{2}\ln(x^2+1) + C$

86. $\displaystyle\int \frac{1}{x\sqrt{6x-x^2}}\,dx = -\frac{\sqrt{6x-x^2}}{3x} + C$

87. $\displaystyle\int \frac{3x+2}{(x^2+4)^{3/2}}\,dx = \frac{x-6}{2\sqrt{x^2+4}} + C$

88. $\displaystyle\int x^{3/2}\ln x\,dx = \frac{2}{5}x^{5/2}\ln x - \frac{4}{25}x^{5/2} + C$

89. $\displaystyle\int \frac{\sqrt{1+\sin^2 x}}{\sec x \csc x}\,dx = \frac{1}{3}(1+\sin^2 x)^{3/2} + C$

90. $\displaystyle\int \frac{e^{\sqrt{\sin x}}}{\sec x\sqrt{\sin x}}\,dx = 2e^{\sqrt{\sin x}} + C$

91. $\displaystyle\int xe^x \sin x\,dx = -\frac{1}{2}e^x(x\cos x - \cos x - x\sin x) + C$

92. $\displaystyle\int x^2 e^{x^{3/2}}\,dx = \frac{2}{3}(x^{3/2}-1)e^{x^{3/2}} + C$

93. $\displaystyle\int \frac{\arctan x}{(x-1)^3}\,dx = -\frac{\tan^{-1} x}{2(x-1)^2} - \frac{1}{4(x-1)} - \frac{1}{4}\ln(x-1) + \frac{1}{8}\ln(x^2+1) + C$

94. $\displaystyle\int \ln(1+\sqrt{x})\,dx = (x-1)\ln(1+sqrtx) + \frac{3}{2} + \sqrt{x} - \frac{x}{2} + C$

95. $\displaystyle\int \frac{2x+3}{\sqrt{3+6x-9x^2}}\,dx = -\frac{2}{9}\sqrt{3+6x-9x^2} + \frac{11}{9}\sin^{-1}\left(\frac{3x-1}{2}\right) + C$

96. $\displaystyle\int \frac{1}{\sqrt{e^{2x}-1}}\,dx = \tan^{-1}\left(\sqrt{e^{2x}-1}\right) + C$

97. $\displaystyle\int \frac{x^4}{(x-1)^2}\,dx = \frac{x^3}{3} + x^2 + 3x - \frac{1}{x-1} + 4\ln(x-1) + C$

98. $\displaystyle\int x^{3/2}\tan^{-1}(\sqrt{x})\,dx = \frac{2}{5}x^{5/2}\tan^{-1}(\sqrt{x}) - \frac{x^2}{10} + \frac{x}{5} - \frac{1}{5}\ln(x+1) + C$

99. $\displaystyle\int \sec^{-1}(\sqrt{x})\,dx = x\sec^{-1}(\sqrt{x}) - \sqrt{x-1} + C$

100. $\displaystyle\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = \frac{1}{2}\sqrt{1-x^4} + \frac{1}{2}\sin^{-1}(x^2) + C$