HOMEWORK 1

Definition (Suspension). The *suspension* of a topological space X is the topological space defined by the set

$$\Sigma X \coloneqq (X \times [0,1])/(X \times \{0,1\}) = (X \times [0,1])/\mathscr{R}$$

of equivalence classes associated to the equivalence relation $\mathscr{R}$ given by

$$(x,s)\mathscr{R}(y,t) \quad \text{if} \quad (x,s)=(y,t) \quad \text{or} \quad s=t=0 \quad \text{or} \quad s=t=1\ ,$$

for $x, y \in X$ and $s, t \in [0,1]$, and equipped with the quotient topology.

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Exercise.

(1) Draw a picture representing the suspension ΣS^1 of the circle.

(2) For any $n \in \mathbb{N}$, give a topological space homeomorphic to the suspension ΣS^n of the n-dimension sphere. (Recall that the 0-dimensional sphere $S^0 = \partial I = \{0,1\}$.)

(3) Prove that the suspension ΣS^n of the n-dimension sphere is homeomorphic to this topological space.