

Homework 2

Definition (Suspension). The *suspension* of a topological space X is the topological space defined by the set

$$\Sigma X := (X \times [0, 1]) / \Re$$

of equivalence classes associated to the equivalence relation ${\mathcal R}$ given by

$$(x, s)\Re(y, t)$$
 if $(x, s) = (y, t)$ or $s = t = 0$ or $s = t = 1$,

for $x, y \in X$ and $s, t \in [0, 1]$, and equipped with the quotient topology.

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Exercise.

- (1) Draw a picture representing the suspension ΣS^1 of the circle.
- (2) For any $n \in \mathbb{N}$, give a topological space homeomorphic to the suspension ΣS^n of the *n*-dimension sphere. (Recall that the 0-dimensional sphere $S^0 = \partial I = \{0, 1\}$.)
- (3) Prove that the suspension ΣS^n of the *n*-dimension sphere is homeomorphic to this topological space.