EXAM 1 - ALGEBRAIC TOPOLOGY

LAST NAME, First name:	Grade:

The exam will last **45** *minutes. No document or electronic device is allowed.*

Exercise 1.

⋄ Classify the letters of the alphabet in capital up to homotopy.

Exercise 2.

⋄ Let $f_0, f_1, g_0, g_1 : [0,1] \to X$ be four paths such that $f_0(0) = f_1(0)$, $f_0(1) = f_1(1) = g_0(0) = g_1(0)$, and $g_0(1) = g_1(1)$. We assume that $f_0 \cdot g_0 \simeq f_1 \cdot g_1$ rel $\{0,1\}$ and $f_0 \simeq f_1$ rel $\{0,1\}$. Show that $g_0 \simeq g_1$ rel $\{0,1\}$.

Exercise 3.

⋄ Let $n \geqslant 2$. Show that $\mathbb{R}^n \setminus \{0\}$ has the same homotopy type than S^{n-1}.

Exercise 4.

⋄ Let $n \in \mathbb{Z}$. We consider the circle as the set of points of norm equal to 1 of the complex plane: $S^1 := \{z \in \mathbb{C} \mid |z| = 1\}$. We consider the continuous map $f\colon S^1 \to S^1$ defined by $f(z) := z^n$. What is the group morphism $\pi_1(f)\colon \pi_1\left(S^1, 1\right) \to \pi_1\left(S^1, 1\right)$ associated to f?

⋄ Show that every group morphism $\pi_1\left(S^1, 1\right) \to \pi_1\left(S^1, 1\right)$ is equal to a morphism $\pi_1(f)$ associated to a continuous map $f\colon S^1 \to S^1$.

Exercise 5.

⋄ We consider the configuration space of 2 points in the plane.

$$\mathrm{Conf}_2\left(\mathbb{R}^2\right) := \left\{(x, y) \in \mathbb{R}^2 \times \mathbb{R}^2 \mid x \neq y\right\} \subset \mathbb{R}^2 \times \mathbb{R}^2 .$$

Which more simple topological space that you know has the same homotopy type?

⋄ What is the fundamental group $\pi_1\left(\mathrm{Conf}_2\right)$ of the configuration space of 2 points in the plane?