EXAM 1 - ALGEBRAIC TOPOLOGY

LAST NAME, First name: Grade:	
-------------------------------	--

The exam will last 45 minutes. No document or electronic device is allowed.

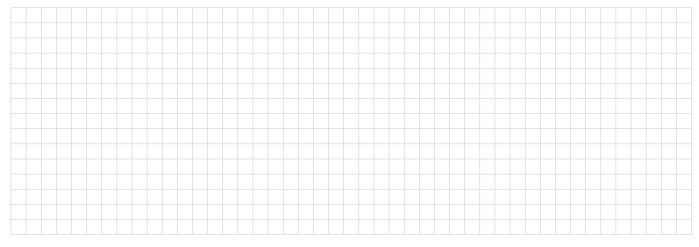
Exercise 1.

♦ Classify the letters of the alphabet in capital up to homotopy.



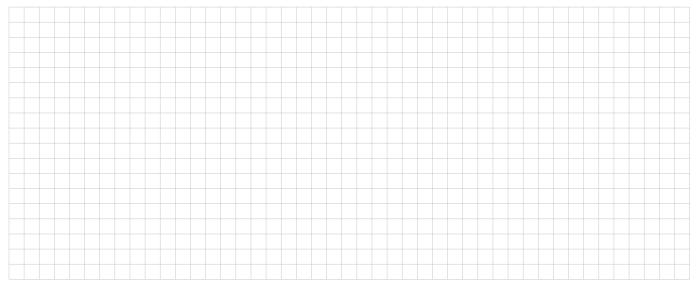
Exercise 2.

♦ Let $f_0, f_1, g_0, g_1 : [0, 1] \to X$ be four paths such that $f_0(0) = f_1(0), f_0(1) = f_1(1) = g_0(0) = g_1(0)$, and $g_0(1) = g_1(1)$. We assume that $f_0 \cdot g_0 \simeq f_1 \cdot g_1$ rel $\{0, 1\}$ and $f_0 \simeq f_1$ rel $\{0, 1\}$. Show that $g_0 \simeq g_1$ rel $\{0, 1\}$.



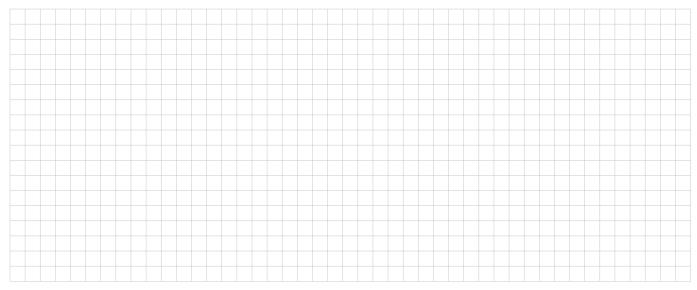
Exercise 3.

 \diamond Let $n \geqslant 2$. Show that $\mathbb{R}^n \setminus \{0\}$ has the same homotopy type than S^{n-1} .



Exercise 4.

 \diamond Let $n \in \mathbb{Z}$. We consider the circle as the set of points of norm equal to 1 of the complex plane: $S^1 := \{z \in \mathbb{C} \mid |z| = 1\}$. We consider the continuous map $f : S^1 \to S^1$ defined by $f(z) := z^n$. What is the group morphism $\pi_1(f) : \pi_1\left(S^1,1\right) \to \pi_1\left(S^1,1\right)$ associated to f?



 \diamond Show that every group morphism $\pi_1(S^1,1) \to \pi_1(S^1,1)$ is equal to a morphism $\pi_1(f)$ associated to a continuous map $f: S^1 \to S^1$.



Exercise 5.

♦ We consider the configuration space of 2 points in the plane.

$$\mathrm{Conf}_2\left(\mathbb{R}^2\right) \coloneqq \left\{(x,y) \in \mathbb{R}^2 \times \mathbb{R}^2 \,|\, x \neq y\right\} \subset \mathbb{R}^2 \times \mathbb{R}^2 \;.$$

Which more simple topological space that you know has the same homotopy type?



 \diamond What is the fundamental group $\pi_1(\operatorname{Conf}_2)$ of the configuration space of 2 points in the plane?

