EXAM 2 - ALGEBRAIC TOPOLOGY

LAST NAME, First name	Grade:

*The exam will last **45** minutes. No document or electronic device is allowed.*

Exercise 1.

⋄ Give the definition of the grassmannian manifolds. How are their topologies defined?

Exercise 2.

⋄ We consider the action of the linear group $\mathrm{GL}_n(\mathbb{R})$, made up of invertible matrices, on $\mathbb{R}^n$ by left multiplication: $(M, X) \mapsto MX$ for $M \in \mathrm{GL}_n(\mathbb{R})$ and $X \in \mathbb{R}^n$. Describe the quotient space $\mathbb{R}^n/\mathrm{GL}_n(\mathbb{R})$: the underlying set and its topology.

Exercise 3.

⋄ Let X be a path-connected topological space and let $\mathcal{R}$ be an equivalence relation on X. Is the quotient space $X/\mathcal{R}$ path-connected? If yes, prove it, if not give a counter-example.

Exercise 4.

◇ Let $n \geqslant 1$. Show that the quotient space B^n/S^{n-1} is homeomorphic to the sphere S^n, where the sphere $S^{n-1} = \partial B^n$ is the boundary of the ball B^n.

Exercise 5.

◇ Let $x \in S^1$ be a point of the circle. Give a topological space homeomorphic to the quotient space

$$\frac{S^1 \times S^1}{S^1 \times \{x\} \cup \{x\} \times S^1}$$

and prove it.

Exercise 6.

◇ Give two equivalent but different definitions of the complex projective space $\mathbb{P}^n(\mathbb{C})$ as a quotient space, for $n \geqslant 0$. Give a cell decomposition of $\mathbb{P}^n(\mathbb{C})$.