EXAM 2 - ALGEBRAIC TOPOLOGY

LAST NAME, First name:	Grade:

The exam will last **45** *minutes. No document or electronic device is allowed.*

Exercise 1.

⋄ Show that the real linear group is homeomorphic to $\mathrm{GL}_n(\mathbb{R}) \cong \mathrm{O}(n) \times \mathbb{R}^{\frac{n(n+1)}{2}}$. (It is not required to prove the continuity of the various maps.)

Exercise 2.

⋄ We consider the following inclusion of the interval into the disc

$$f\colon I = [0,1] \to D = \left\{(x,y) \in \mathrm{R}^2 \mid x^2 + y^2 \leqslant 1\right\} \quad \text{defined by} \quad f(x) \coloneqq (x,0).$$

Draw the pushout P defined by

$$\begin{array}{ccc} I & \xrightarrow{f} & D \\ {\scriptstyle i_0}\downarrow & & \downarrow \\ I \times I & \longrightarrow & P, \end{array} \qquad \text{where} \quad i_0(x) \coloneqq (x,0).$$

and give it a cellular decomposition. (No proof is required.)

Exercise 3.

⋄ We consider the equivalence relation on $\mathbb{R}$ defined by: $x \sim y$ si $y - x \in \mathbb{Q}$. What are the open sets of the quotient topology on $\mathbb{R}/\sim$?

Exercise 4.

⋄ We consider the transitive topological action of the special linear group $\mathrm{SL}_2(\mathbb{R})$ on the Poincaré half-plane $H := \{z \in \mathbb{C} \mid \mathrm{Im} z > 0\}$ defined by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot z := \frac{az + b}{cz + d} .$$

Show that the stabilizer of any element $z \in H$ is isomorphic to the topological group S^1.

⋄ Show that the Poincaré half-plane is homeomorphic to $H \cong \mathrm{SL}_2(\mathbb{R})/\mathrm{SO}(2)$.