EXAM 2 - ALGEBRAIC TOPOLOGY

LAST NAME, First name: Grade:

The exam will last 45 minutes. No document or electronic device is allowed.

Exercise 1.

 \diamond Show that the real linear group is homeomorphic to $\mathrm{GL}_n(\mathbb{R}) \cong \mathrm{O}(n) \times \mathbb{R}^{\frac{n(n+1)}{2}}$. (It is not required to prove the continuity of the various maps.)



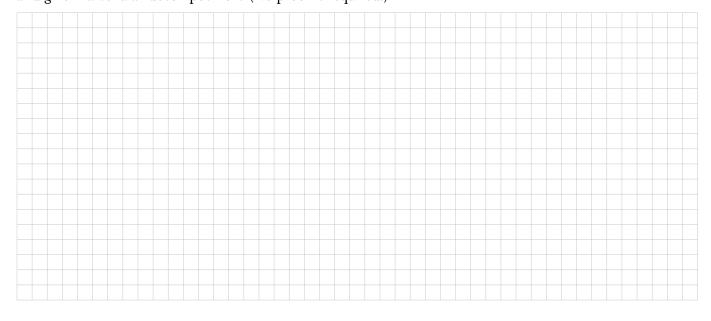
Exercise 2.

♦ We consider the following inclusion of the interval into the disc

$$f: I = [0, 1] \to D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \le 1\}$$
 defined by $f(x) := (x, 0)$.

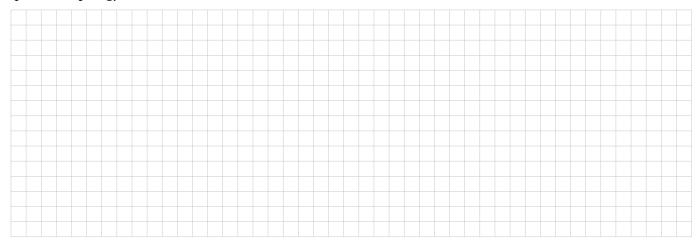
Draw the pushout P defined by

and give it a cellular decomposition. (No proof is required.)



Exercise 3.

 \diamond We consider the equivalence relation on $\mathbb R$ defined by: $x \sim y$ si $y - x \in \mathbb Q$. What are the open sets of the quotient topology on $\mathbb R/\sim$?

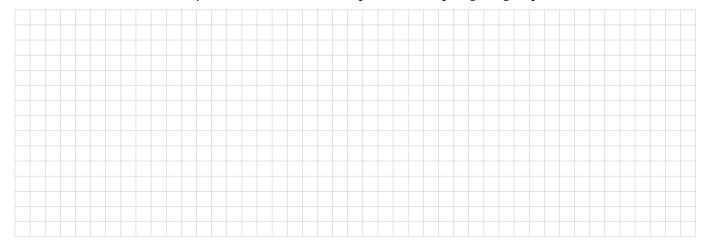


Exercise 4.

 \diamond We consider the transitive topological action of the special linear group $\mathrm{SL}_2(\mathbb{R})$ on the Poincaré half-plane $H:=\{z\in\mathbb{C}\mid \mathrm{Im} z>0\}$ defined by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot z \coloneqq \frac{az+b}{cz+d} \ .$$

Show that the stabilizer of any element $z \in H$ is isomorphic to the topological group S^1 .



 \diamond Show that the Poincaré half-plane is homeomorphic to $H \cong \mathrm{SL}_2(\mathbb{R})/\mathrm{SO}(2)$.

