EXAM 3 - ALGEBRAIC TOPOLOGY

LAST NAME, First name:	Grade:

The exam will last **45** *minutes. No document or electronic device is allowed.*

Exercise 1.

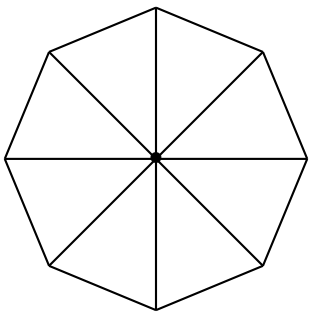
◇ What is the definition of a topological manifold of dimension $n \in \mathbb{N}$? Give an example in dimension 3.

Exercise 2.

◇ Give a cellular decomposition of the projective space $\mathbb{P}^n\mathbb{R}$, for $n \in \mathbb{N}$, and prove it.

Exercise 3.

◇ Let S be a surface defined by a planar representation given by a polygon P with $2n$ edges. We consider the construction which consists in placing a point at the center of the polygon and drawing all the edges from this point to the vertices of the polygon P. Does this form a triangulation of S ? If yes, prove it, otherwise give a counter-example.



Exercise 4.

◇ Give a triangulation of the Klein bottle. (A picture is enough.)

◇ Compute the Euler characteristic $\chi(\mathrm{Klein})$ of the Klein bottle.

Exercise 5.

◇ To which space is homeomorphic the connected sum $\mathbb{P}^2\mathbb{R}\#\mathbb{P}^2\mathbb{R}$? Prove it.