Homotopy theory of Batalin–Vilkovisky algebras

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Plan

Homotopy data and mixed complex structure

- **Homotopy data:** Deformation retract of chain complexes

$$h \circlearrowright (A, d_A) \underset{i}{\overset{p}{\rightleftarrows}} (H, d_H) \qquad \boxed{\mathrm{id}_A - ip = d_A h + h d_A}\ .$$

- **Algebraic data:** $\Delta : A \to A, \quad d_A\Delta + \Delta d_A = 0, \quad \boxed{\Delta^2 = 0}$
  mixed complex $|\Delta| = 1$ (or bicomplex).

- **Transferred structure:** $\boxed{\delta_1 := p\Delta i}$

  Does δ_1 squares to zero?

$$(\delta_1)^2 = p\Delta \underbrace{ip}_{\sim_h \mathrm{id}_A} \Delta i \neq 0 \text{ in general!}$$

  Idea: Introduce $\boxed{\delta_2 := p\Delta h \Delta i}$

  Then, $\boxed{\partial(\delta_2) = (\delta_1)^2}$ in $(\mathrm{Hom}(A, A), \partial := [d_A, -])$.

  $\Longrightarrow$ δ_2 is a homotopy for the relation $(\delta_1)^2 = 0$.

Higher structure: multicomplex

Higher up, we consider: $\boxed{\delta_n := p(\Delta h)^{n-1}\Delta i}$, for $n \geq 1$.

Proposition

$$\boxed{\partial(\delta_n) = \sum_{k=1}^{n-1} \delta_k \delta_{n-k}} \quad \textit{in } (\mathrm{Hom}(A, A), \partial), \ \textit{for} \ \ n \geq 1 \ .$$

Definition (Multicomplex)

$(H, \delta_0 := -d_H, \delta_1, \delta_2, \ldots)$ graded vector space H endowed with a family of linear operators of degree $|\delta_n| = 2n - 1$ satisfying

$$\boxed{\sum_{k=0}^{n} \delta_k \delta_{n-k} = 0} \ , \quad \text{for} \ \ n \geq 0 \ .$$

Remark: A mixed complex = multicomplex s.t. $\delta_n = 0$, for $n \geq 2$.

Multicomplexes are homotopy stable

- Starting now from a multicomplex $(A, \Delta_0 = -d_A, \Delta_1, \Delta_2, \ldots)$
- Consider the transferred operators

$$\boxed{\delta_n := \sum_{k_1+\cdots+k_l=n} p\Delta_{k_1} h\Delta_{k_2} h \ldots h\Delta_{k_l} i}\ , \quad \text{for} \quad n \geq 1$$

Proposition

$$\boxed{\partial(\delta_n) = \sum_{k=1}^{n-1} \delta_k \delta_{n-k}} \quad \textit{in } (\mathrm{Hom}(A, A), \partial), \ \textit{for} \quad n \geq 1 \ .$$

$\Longrightarrow$ Again a multicomplex, **no need of further higher structure**.

Compatibility between Original and Transferred structures

$$\underbrace{(A, \Delta_0 = -d_A, \Delta_1, \Delta_2, \ldots)}_{\text{Original structure}} \xleftarrow{\ i\ } \underbrace{(H, \delta_0 = -d_H, \delta_1, \delta_2, \ldots)}_{\text{Transferred structure}}$$

- i chain map $\implies$ $\boxed{\Delta_0 i = i \delta_0}$
- **Question:** Does i commute with the Δ's and the δ's?

$$i\delta_1 = \underbrace{ip}_{\sim_h \ \mathrm{id}_A} \Delta_1 i \neq \Delta_1 i \quad \text{in general!}$$

- Define $\boxed{i_0 := i}$ and consider $\boxed{i_1 := h\Delta_1 i}$.

  Then, $\boxed{\partial(i_1) = \Delta_1 i_0 - i_0 \delta_1}$ in $(\mathrm{Hom}(H, A), \partial)$.

  $\implies$ i_1 is a homotopy for the relation $\Delta_1 i_0 = i_0 \delta_1$.

∞-morphisms of multicomplexes

Higher up, we consider:

$$\boxed{i_n := \sum_{k_1+\cdots+k_l=n} h\Delta_{k_1} h\Delta_{k_2} h \ldots h\Delta_{k_l} i} \ , \quad \text{for} \quad n \geq 1 \ .$$

$$\Rightarrow \boxed{\partial(i_n) = \sum_{k=0}^{n-1} \Delta_{n-k} i_k - \sum_{k=0}^{n-1} i_k \delta_{n-k}} \ \text{in } (\mathrm{Hom}(H, A), \partial), \ \text{for } n \geq 1 \ .$$

Definition (∞-morphism)

$i_\infty : (H, \delta_0 = -d_H, \delta_1, \delta_2, \ldots) \rightsquigarrow (A, \Delta_0 = -d_A, \Delta_1, \Delta_2, \ldots)$
collection of maps $\{i_n \colon H \to A\}_{n \geq 0}$ satisfying

$$\boxed{\sum_{k=0}^{n} \Delta_{n-k} i_k = \sum_{k=0}^{n} i_k \delta_{n-k}} \ , \quad \text{for} \quad n \geq 0 \ .$$

The category ∞-mutlicomp

Proposition (Composite of ∞-morphisms)

$f : A \rightsquigarrow B$, $g : B \rightsquigarrow C$: two ∞-morphisms of multicomplexes.

$$\boxed{(gf)_n := \sum_{k=0}^{n} g_{n-k} f_k}\ , \quad \textit{for} \quad n \geq 0\ ,$$

defines an associative and unital composite of ∞-morphisms.

Category: multicomplex with ∞-morphisms: ∞-multicomp.

multicomplex = square-zero element

$$\Delta(z) = \Delta_0 + \Delta_1 z + \Delta_2 z^2 + \cdots$$

in the algebra $\mathrm{End}_A[[z]]$,
∞-morphism $= i(z) \in \mathrm{Hom}(H, A)[[z]]$ s.t. $i(z)\delta(z) = \Delta(z)i(z)$,
composite $= g(z)f(z)$.

Homotopy Transfer Theorem for multicomplexes

∞**-quasi-isomorphism:** $i : H \overset{\sim}{\rightsquigarrow} A$ s.t. $i_0 : H \xrightarrow{\sim} A$ qi.

Theorem (HTT for multicomplexes, Lapin '01)

Given any deformation retract

$$h \circlearrowright (A, d_A) \underset{i}{\overset{p}{\rightleftarrows}} (H, d_H) \qquad \mathrm{id}_A - ip = d_A h + h d_A$$

and any multicomplex structure on A, there exists a multicomplex structure on H such that i extends to an ∞-quasi-isomorphism.

Application 1: $\mathbb{K}$ field, (A, d, Δ) bicomplex, $(H(A, d), 0) = E^1$ deformation retract

$\Longrightarrow$ multicomplex structure on $H(A, d)$ =lift of the spectral sequence, i.e. $\delta^r \Rightarrow d^r$.

Application 2: Equivalence between the various definitions of cyclic homology [Loday-Quillen, Kassel].

Homotopy theory of mixed complexes

Proposition (Rectification)

$\exists$ $Rect$: ∞-multicomp $\to$ mixed cx, *s.t.*

$$H \overset{\sim}{\rightsquigarrow} Rect(H) := (\underbrace{H[[z]] \oplus H[[z]]\Delta}_{\textit{free mixed complex}}, d).$$

Application (HTT): $Rect((H, \delta_0, \delta_1, \ldots)) \cong A$ in Ho(mixed cx).

Definition (Homotopy relation)

$\exists$ notion of homotopy between ∞-morphisms: $\sim_h$.

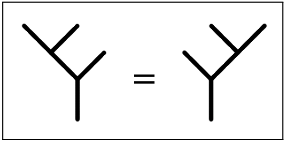
Theorem (?)

- *Every ∞-qi of multicomplexes admits a homotopy inverse.*
- Ho(mixed cx) $:=$ mixed cx $[qi^{-1}] \cong \infty$-mixed cx$/ \sim_h$.

Associative algebra and homotopy data

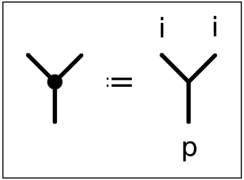
- **Initial structure:** an associative product on A

$$\nu : A^{\otimes 2} \to A, \quad \text{s.t.} \quad \nu(\nu(a,b),c) = \nu(a,\nu(b,c)) \ .$$



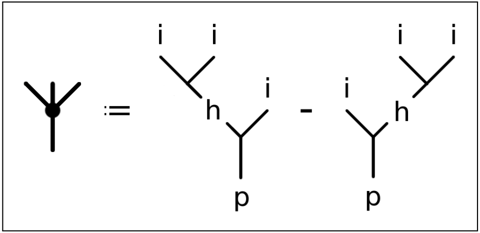
- **Transferred structure:** the binary product on H

$$\mu_2 := p\nu i^{\otimes 2} \ : \ H^{\otimes 2} \to H.$$

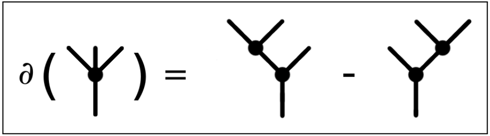


First homotopy for the associativity relation

- Is the transferred μ_2 associative? Anwser: in general, no!
- Introduce μ_3:



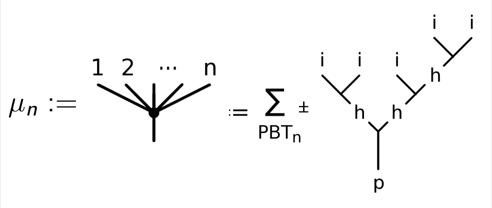
- In $\mathrm{Hom}(A^{\otimes 3}, A)$, it satisfies



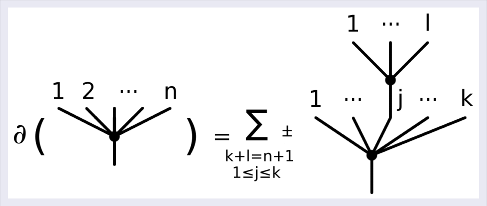
$\Longrightarrow$ μ_3 is a homotopy for the associativity relation of μ_2.

Higher structure

Higher up, in $\mathrm{Hom}(H^{\otimes n}, H)$, we consider:



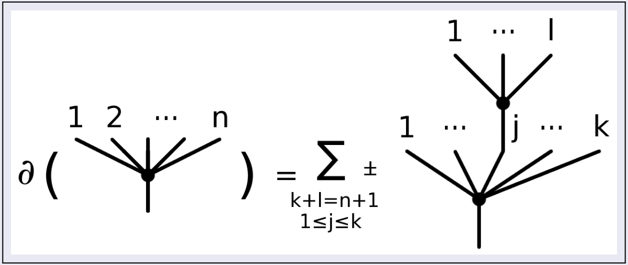
Proposition



A_∞-algebra

Definition (A_∞-algebra, Stasheff '63)

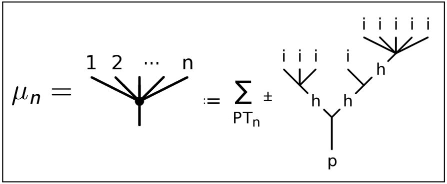
An A_∞-algebra is a chain complex $(H, d_H, \mu_2, \mu_3, \ldots)$ endowed with a family of multlinear maps of degree $|\mu_n| = n - 2$ satisfying



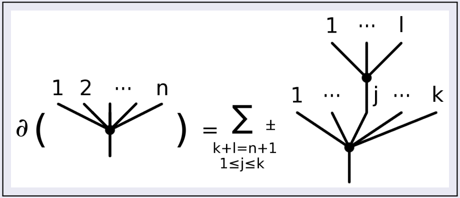
Remark: A dga algebra $=$ A_∞-algebra s.t. $\mu_n = 0$, for $n \geq 3$.

A_∞-algebras are homotopy stable

- Starting from an A_∞-algebra $(A, d_A, \nu_2, \nu_3, \ldots)$
- Consider



Proposition



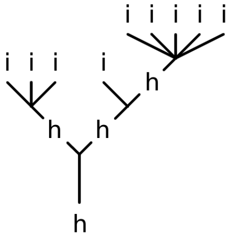
$\Longrightarrow$ Again an A_∞-algebra, **no need of further higher structure**.

Compatibility between Original and Transferred structures

$$\underbrace{(A, d_A, \nu_2, \nu_3, \ldots)}_{\text{Original structure}} \xleftarrow{\ i\ } \underbrace{(H, d_H, \mu_2, \mu_3, \ldots)}_{\text{Transferred structure}}$$

- i chain map $\Longrightarrow$ $\boxed{d_A i = i d_H}$
- **Question:** Does i commutes with the ν's and the μ's?
  Anwser: not in general!
- Define $\boxed{i_1 := i}$ and consider in $\mathrm{Hom}(H^{\otimes n}, A)$, for $n \geq 2$:

$$i_n := \sum_{\mathrm{PT}_n} \pm$$



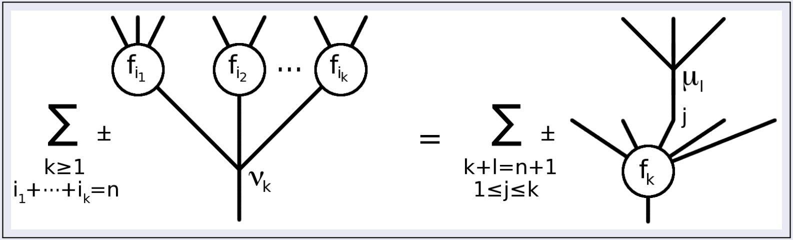
A_∞-morphism

Definition (A_∞-morphism)

$(A, d_A, \{\mu_n\}_{n\geq 2}) \rightsquigarrow (B,\ d_B,\ \{\nu_n\}_{n\geq 2})$ is a collection of linear maps

$$\{f_n\ :\ A^{\otimes n} \longrightarrow B\}_{n\geq 1}$$

of degree $|f_n| = n-1$ satisfying



Example: The aforementioned $\{i_n\ :\ H^{\otimes n} \longrightarrow A\}_{n\geq 1}$.

The category ∞-A_∞-Alg

Category: A_∞-algebras with ∞-morphisms: ∞-A_∞-Alg.

A_∞-algebra $=$ square-zero coderivation in the coalgebra $T^c(sA)$,
A_∞-morphism $=$ morphism of dg coalgebras $T^c(sA) \to T^c(sB)$.

Definition (∞-quasi-isomorphism)

$$i : H \stackrel{\sim}{\rightsquigarrow} A \quad \text{s.t.} \quad i_1 : H \xrightarrow{\sim} A \text{ is a quasi-isomorphism.}$$

Proposition (Rectification)

$$\exists\ Rect : \infty\text{-}A_\infty\text{-Alg} \to \text{dga alg},\ s.t.\ \ H \stackrel{\sim}{\rightsquigarrow} Rect(H).$$

Theorem (Munkholm '78, Lefèvre-Hasegawa '03)

- *Every ∞-qi of A_∞-algebras admits a homotopy inverse.*
- $\mathrm{Ho}(\text{dga alg}) := \text{dga alg}\,[qi^{-1}] \cong \infty\text{-dga alg}/ \sim_h$.

Homotopy Transfer Theorem for A_∞-algebras

Theorem (HTT for A_∞-algebras, Kadeshvili '82)

Given any deformation retract

$$h \circlearrowright (A, d_A) \underset{i}{\overset{p}{\rightleftarrows}} (H, d_H) \qquad \mathrm{id}_A - ip = d_A h + h d_A$$

and any A_∞-algebra structure on A, there exists an A_∞-algebra structure on H such that i extends to an ∞-quasi-isomorphism.

Alternative: $Rect(H) \cong A$ in Ho(dga alg).

Application: $A = (C^\bullet_{\mathrm{Sing}}(X), \cup)$, transferred A_∞-algebra on $H^\bullet_{\mathrm{Sing}}(X)$= lifting of the (higher) Massey products.

Homotopy algebra and operads

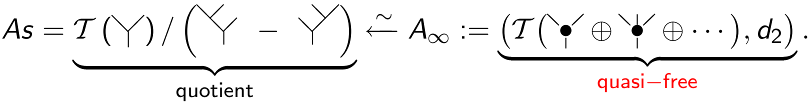
$$\begin{tikzcd}
\text{operad } \mathcal{P} \arrow[d, dotted] & & \mathcal{P}_\infty \arrow[ll, "\sim"'] \arrow[d, dotted] & \text{: quasi-free replacement (cofibrant)} \\
\text{category of } \mathcal{P}\text{-algebras} \arrow[rr, hook] & & \text{category of homotopy } \mathcal{P}\text{-algebras}
\end{tikzcd}$$

Examples:

- $\mathcal{P} = D$: D_∞-algebras $=$ multicomplexes

$$D = \underbrace{T(\Delta)/(\Delta^2)}_{\text{quotient}} \xleftarrow{\sim} D_\infty := \underbrace{\left(T(\delta \oplus \delta^2 \oplus \delta^3 \oplus \cdots), d_2\right)}_{\text{quasi}-\text{free}}.$$

- $\mathcal{P} = Ass$: Ass_∞-algebras $=$ A_∞-algebras



Koszul duality theory

$$\boxed{\mathcal{P}_\infty = \mathcal{T}(\text{operadic syzygies}) \xrightarrow{?\sim?} \mathcal{P}}$$

- **Quadratic presentation:** $\mathcal{P} = \mathcal{T}(V)/(R)$, where
$$R \subset \underbrace{\mathcal{T}^{(2)}(V)}_{\text{trees with 2 vertices}} .$$
- **Koszul dual cooperad:** quadratic cooperad $\mathcal{P}^{¡} := \mathcal{C}(sV, s^2R)$, i.e. defined by a (dual) universal property.
- **Candidate:** $\mathcal{P}_\infty = \Omega\mathcal{P}^{¡} = \mathcal{T}(\mathcal{P}^{¡}) \xrightarrow{?\sim?} \mathcal{P}$.
- **Criterion:** Quasi-isomorphism iff the Koszul complex $\mathcal{P} \circ_\kappa \mathcal{P}^{¡}$ is acyclic.
- **Examples:** D, Ass, Com, Lie, $Gerst$, etc.

Operadic higher structure

For any Koszul operad $\mathcal{P}$

- $\exists$ a notion of composable ∞-morphisms: ∞-$\mathcal{P}_\infty$-Alg.

$\mathcal{P}_\infty$-algebra $=$ square-zero coderivation in the coalgebra $\mathcal{P}^{\text{i}}(A)$,
∞-morphism $=$ morphism of dg coalgebras $\mathcal{P}^{\text{i}}(A) \to \mathcal{P}^{\text{i}}(B)$.

Theorem (HTT for P_∞-algebras, Galvez–Tonks-V.)

Given any deformation retract

$$h \circlearrowright (A, d_A) \underset{i}{\overset{p}{\rightleftarrows}} (H, d_H) \qquad \mathrm{id}_A - ip = d_A h + h d_A$$

and any $\mathcal{P}_\infty$-algebra structure on A, there exists an $\mathcal{P}_\infty$-algebra structure on H such that i extends to an ∞-quasi-isomorphism.

"Application": [wheeled properads, Merkulov '10]
perturbation theory in QFT $=$ HTT for involutive Lie bialgebras:
Feynman diagrams $=$ Graphs formulae for transferred structure.

Batalin–Vilkovisky algebras

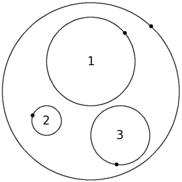
Definition (Batalin–Vilkovisky algebra)

Graded commutative algebra $(A, d_A, \cdot)$ endowed with a linear operator $\Delta^2 = 0$, $d_A\Delta + d_A\Delta = 0$, of order 2:

$$\Delta(abc) = \Delta(ab)c + \Delta(bc)a + \Delta(ca)b - \Delta(a)bc - \Delta(b)ca - \Delta(c)ab\,.$$

Examples : $H_\bullet(TCFT)$, $\mathbb{H}_\bullet(\mathcal{L}X)$ (string topology), Dolbeault complex of Calabi-Yau manifolds, etc.

Operadic topological interpretation: $H_\bullet(fD_2) = BV$.



Open questions

- Find a quasi-free resolution $BV_\infty = \mathcal{T}(?) \xrightarrow{\sim} BV$.
- Describe the homotopy category

$$\mathrm{Ho}(\mathrm{dg}\ BV\text{-alg}) := \mathrm{dg}\ BV\text{-alg}\,[qi^{-1}] \cong ?\ .$$

- Understand (and generalize)

Theorem (Barannikov–Kontsevich–Manin)

$(A, d, \cdot, \Delta)$ *dg BV-algebra satisfying the $d\Delta$-lemma*

$$\ker d \cap \ker \Delta \cap (\mathrm{Im} d + \mathrm{Im}\Delta) = \mathrm{Im}(d\Delta) = \mathrm{Im}(\Delta d)$$

$\Longrightarrow$ $H_\bullet(A, d)$ *carries a Frobenius manifold structure, which extends the transferred commutative product.*

Homotopy BV-algebras

Theorem (Galvez–Tonks–V.)

The inhomogeneous Koszul duality theory provides us with a quasi-free resolution $BV_\infty := \Omega BV^{i} \xrightarrow{\sim} BV$.

Proof. In any BV-algebra

$$[\text{-},\text{-}] \;:=\; \Delta \circ (\text{-} \cdot \text{-}) \;-\; (\Delta(\text{-}) \cdot \text{-}) \;-\; (\text{-} \cdot \Delta(\text{-}))$$

is a degree 1 Lie bracket $\Longrightarrow$ new presentation of the operad BV:

$$\begin{aligned} BV \;&\cong\; \mathcal{T}(\cdot, \Delta)/(\text{homogeneous quadratic and cubical relations}) \\ &\cong\; \mathcal{T}(\cdot, \Delta, [\,,\,])/(\text{inhomogeneous quadratic relations})\ . \end{aligned}$$

$\square$

Application: Notion of homotopy BV algebras & ∞-morphisms.

Category: ∞-BV_∞-Alg

Homotopy category

Theorem (V.)

$$\mathrm{Ho}(\text{dg } BV\text{-alg}) := \text{dg } BV\text{-alg}\,[qi^{-1}] \cong \infty\text{-dg } BV\text{-alg}/\sim_h .$$

Proof.

$$\begin{tikzcd}
\text{dg } BV\text{-alg} \arrow[r, shift right, "\mathrm{B}_\kappa"'] \arrow[d, hook] & \text{conil dg } BV^{¡}\text{-coalg} \arrow[l, shift right, "\Omega_\kappa"'] \\
\infty\text{-}BV_\infty\text{-alg} \arrow[u, bend left=40, "\mathrm{Rect}"] \arrow[r, "\cong"] & \text{quasi-free } BV^{¡}\text{-coalg} \arrow[u, hook]
\end{tikzcd}$$

Model category structures: conil dg $BV^{¡}$-coalg (we $\subsetneq$ qi)& ∞-BV_∞-alg (without equalizers). ☐

Generalization: Works for any inhomogeneous Koszul operad $\mathcal{P}$ (after Quillen, Hinich, Lefèvre-Hasegawa).

Applications in Mathematical Physics and in Geometry

Application 1: Lian–Zuckerman conjecture for Topological Vertex Operator Algebra.

Theorem (Lian–Zuckerman '93)

$$H^\bullet_{BRST}(TVOA) : BV\text{-}algebra.$$

Theorem (Lian–Zuckerman conjecture, Galvez–Tonks–V)

$C^\bullet_{BRST}(TVOA) = TVOA$: **explicit** *BV_∞-algebra, which lifts the Lian–Zuckerman operations.*

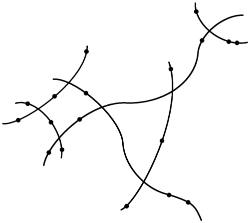
Remarks:

- Lian–Zuckerman conjecture similar to the Deligne conjecture.
- Conjecture: some converse should be true, i.e. $BV_\infty \cong TVOA$.

Application 2: [Dotsenko–Shadrin–V.]
Description of Givental stabilizers of Cohomological Field Theories.

Frobenius manifold

Deligne–Mumford moduli space of stable curves: $\overline{\mathcal{M}}_{g,n+1}$



Definition (Frobenius manifold)

Algebra over $H_\bullet(\overline{\mathcal{M}}_{0,n+1})$, i.e. $H_\bullet(\overline{\mathcal{M}}_{0,n+1}) \to \mathrm{End}_{H_\bullet(A)} \Longleftrightarrow$ totally symmetric n-ary operation $(x_1, \ldots, x_n)$ of degree $2(n-2)$,

$$\sum_{S_1 \sqcup S_2 = \{1,\ldots,n\}} ((a, b, x_{S_1}), c, x_{S_2}) = \sum_{S_1 \sqcup S_2 = \{1,\ldots,n\}} \pm(a, (b, x_{S_1}, c), x_{S_2}) \, .$$

Topological interpretation: homotopy trivialization of S^1

Application 3: Barannikov–Kontsevich–Manin Theorem ???

$$BV^{¡} \cong T^c(\delta) \otimes Com_1^* \circ Lie^* \xleftrightarrow{???} H_\bullet(\overline{\mathcal{M}}_{0,n+1}),\ \text{so, not yet!}$$

Conjecture: [Costello–Kontsevich] $fD_2 \,/_h\, S^1 \cong \overline{\mathcal{M}}_{0,n+1}$.

Theorem (Drummond-Cole – V.)

Minimal model of BV : $\mathcal{T}\big(T^c(\delta) \oplus H^{\bullet+1}(\mathcal{M}_{0,n+1})\big) \xrightarrow{\sim} BV$.

Homotopy trivialization of the circle $\Longleftrightarrow$ trivial action of $T^c(\delta)$

$$H_\bullet(\overline{\mathcal{M}}_{0,n+1})^{¡} = H^{\bullet+1}(\mathcal{M}_{0,n+1}) \ \&\ \text{Koszul} \qquad \text{[Getzler '95]}$$

Solution of the conjecture over $\mathbb{Q}$

$$BV_\infty \,/_h\, \Delta = \underbrace{\mathcal{T}(H^{\bullet+1}(\mathcal{M}_{0,n+1}))}_{\text{homotopy Frobenius manifold}} \xrightarrow{\sim} \underbrace{H_\bullet(\overline{\mathcal{M}}_{0,n+1})}_{\text{Frobenius manifold}} .$$

HTT for homotopy BV-algebras with Δ trivialization

[BKM]: $(A, d, \cdot, \Delta)$ dg BV-algebra satisfying the $d\Delta$-lemma $\Longrightarrow$ $H_\bullet(A, d)$ carries a Frobenius manifold structure.

Theorem (Drummond-Cole – V.)

$(A, d, \cdot, \Delta)$ dg BV-algebra satisfying the Hodge–de Rham condition $\Longrightarrow$ $H_\bullet(A, d)$ carries a homotopy Frobenius manifold structure, which extends the Frobenius manifold structure and

$$Rect(H_\bullet(A), d) \sim (A, d, \cdot, \Delta) \text{ in } \mathrm{Ho}(dg\ BV\text{-}alg)$$

$$\begin{tikzcd} H_\bullet(\overline{\mathcal{M}}_{0,n+1}) \arrow[r, "{[BKM]}"] & \mathrm{End}_{H_\bullet(A)} \\ H^{\bullet+1}(\mathcal{M}_{0,n+1}) \arrow[u, dotted, "\kappa"] \arrow[ur, dotted, "{[DCV]}"'] & \end{tikzcd}$$

De Rham cohomology of Poisson manifolds

Theorem (Koszul '85)

(M, π) Poisson manifold $\Longrightarrow$ De Rham complex

$$(\Omega^\bullet M, d_{DR}, \wedge, \Delta := [i_\pi, d_{DR}]): \ \textit{BV-algebra.}$$

Theorem (Merkulov '98): M symplectic manifold satisfying the Hard Lefschetz condition $\Longrightarrow$ $H^\bullet_{DR}(M)$: Frobenius manifold.

Theorem (Dotsenko–Shadrin–V.)

For any Poisson manifold $M \Longrightarrow$ $H^\bullet_{DR}(M)$: homotopy Frobenius manifold , s.t.

$$Rect\left(H_\bullet^{DR}(M)\right) \sim (\Omega^\bullet M, d_{DR}, \wedge, \Delta) \ \textit{in } \mathrm{Ho}(\textit{dg BV-alg}).$$

Generalization: (M, π, E) Jacobi manifold (eg contact),

$(\Omega^\bullet \mathcal{M}, d_{DR}, \wedge, \Delta_1 := [i_\pi, d_{DR}], \Delta_2 := i_\pi i_E)$: BV_∞-algebra.

Conjectures in Mirror symmetry

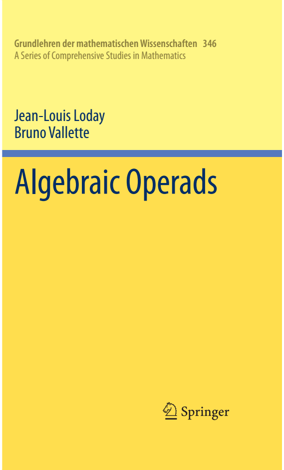
A-side		B-side
Symplectic geometry	$\leftrightarrow$	Complex geometry
$(\Omega^{n-\bullet}(M), d_{DR}, \wedge, \Delta)$	$\stackrel{\sim}{\rightsquigarrow}$	$(\Gamma(\widetilde{M}, \wedge^\bullet \bar{T}^*_{\widetilde{M}} \otimes \wedge^\bullet T_{\widetilde{M}}), \bar{\partial}, \wedge, \mathrm{div})$
$H^{n-\bullet}_{DR}(M)$	$\leftrightarrow$	$H^\bullet_{Dolbeault}(\widetilde{M})$
homotopy Frobenius manifold		homotopy Frobenius manifold
M compact symplectic $\Rightarrow$		$\widetilde{M}$ Calabi-Yau $\Rightarrow$
Formality: Frobenius manifold		Formality: Frobenius manifold

$\exists$ ∞-quasi-isomorphism of dg BV-algebras $\Longrightarrow$ the deformation functors associated to the two Master equations

$$d\alpha + \hbar\Delta\alpha + \frac{1}{2}[\alpha, \alpha] = 0$$

should be isomorphic.

http://math.unice.fr/∼brunov/Operads.pdf



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