OPERADS 2009

CIRM, 27-30 April 2009

Sergei Barannikov (Paris) *Quantum A-infinity algebras, compactified moduli spaces and supersymmetric matrix integrals*

ABSTRACT: Quantum A-infinity algebras are defined by the equation

$$\Delta m + \frac{1}{2}\{m, m\} = 0$$

where $m \in \oplus_n (A^{\otimes n} \otimes k[S_n])^{S_n}$, and S_n is the group of permutations. Such algebras are the generalization of A-infinity algebras which arise in symplectic geometry when counting of holomorphic disks (=genus zero curve with one hole) is extended to counting of curves of arbitrary genus with arbitrary number of holes. The operad underlying quantum A_∞−algebras is the Feynman transform of the modular operad, whose operations are marked by arbitrary permutations: $P((n)) = k[S_n]$ (the subspace of cyclic permutations is the cyclic operad of associative algebras). I have proven in 2005 that the Feynman transform of $P((n))$ is identified, as a complex, with the complex computing cohomology of the Kontsevich compactification of moduli spaces of curves. The consequence of this is the construction of characteristic classes of algebras over $P((n))$ and of algebras over the Feynman transform of $P((n))$, whose values lie in the cohomology/homology of the Kontsevich compactification. The quantum A-infinity algebras are intimately related with integration on (super)matrix spaces and the characteristic classes of quantum A-infinity algebras are terms of the asymptotic expansion of associated supersymmetric matrix integrals.

Michael Batanin (Sydney) *Higher braided operads and stabilization hypotheses.*

ABSTRACT: The Breen-Baez-Dolan stabilization hypothesis predicts that the category of n-monoidal k-category is equivalent to the category of $(n+1)$-monoidal k-category for all $n > k+1$. We deduce this hypothesis from the following stabilization theorem: The model category of n-braided operads in a k-truncated symmetric monoidal model category V is Quillen equivalent to the model category of $(n+1)$-braided operads in V if $n > k+1$. This is joint work with Clemens Berger.

Clemens Berger (Nice) *The cyclic Deligne conjecture for spaces, chain complexes and Hopf algebras.*

ABSTRACT: We show that the Hochschild cochain complex of a symmetric Frobenius algebra comes equipped with a natural action by a cellular model of the framed little disks operad. This action is induced by a general property of multiplicative cyclic operads. The latter, suitably interpreted, recovers Connes-Moscovici's cyclic cohomology of quasi-involutive Hopf algebras. (This is joint work with Michael Batanin).

Alexander Berglund (Copenhaguen) *Homological perturbation theory for algebras over operads.*

ABSTRACT: Homological perturbation theory is a tool in algebraic topology and homological algebra that originates from work of Brown and Gugenheim, and that has been developed subsequently by Gugenheim, Lambe, Stasheff and others. One of the first uses was to give a model for the chains of the total space in a fibration as a 'twisted tensor product' of the chains of the base and the fiber.

The goal of this talk is to show how homological perturbation theory can be extended to encompass extra algebraic structures on chain complexes, such as commutative multiplications, Lie brackets or, more generally, algebra structures over operads. This solves a problem that was pointed out by Gugenheim, Lambe and Stasheff.

As an application, this provides a systematic way of deriving explicit formulas for transferring, for instance, L_∞-algebra structures along quasi-isomorphisms of chain complexes.

Emily Burgunder (IPDE, Varsovie) *The Hopf algebra of parking functions*

ABSTRACT: Parking functions are combinatorial objects defined as a sequence of nonnegative integers majorated by a permutation of 1,...,n. We show that the tridendriform structure on the parking functions, defined by Novelli-Thibon, can be induced from a family of shuffle bialgebras on some combinatorial Hopf algebras. Moreover, we show that tridendriform bialgebras admit a structure theorem, that is to say: a cofree tridendriform bialgebra can be understood as an enveloping algebra over its primitives. Then, we unravel the structure of the primitives, which is a variation of a Gerstenhaber-Voronov algebra. We apply this to the algebra of parking functions to show that it is free as tridendriform algebra. This is a joint work with Maria Ronco.

Vladimir Dotsenko (Dublin) *Groebner bases for operads.*

ABSTRACT: We define a new monoidal structure on (nonsymmetric) collections — shuffle composition of collections. Monoids in the

category of collections with this structure (shuffle operads) turn out to bring a new insight in the theory of symmetric operads. For this category, we develop the machinery of Groebner bases for operads, and present operadic versions of Bergman's Diamond Lemma and Buchberger's algorithm. These results can be applied to symmetric operads; in particular, we obtain an effective algorithmic criterion of Koszulness for (symmetric) quadratic operads. The talk is based on a joint work with A.Khoroshkin.

Joachim Kock (Barcelona) *Feynman graphs, and nerve theorem for modular operads.*

Abstract: I will describe a category of Feynman graphs Φ with the features needed to get the following nerve theorem: coloured modular operads (in Set) are presheaves on Φ satisfying a Segal condition. This is joint work with André Joyal.

Pascal Lambrechts (Louvain-la-neuve) *Formality of the little disk operad and spaces of smooth embeddings.*

Abstract: Let M be a smooth compact manifold and consider the space

$$\mathrm{Emb}(M, \mathbb{R}^d)$$

consisting of all smooth embeddings of M into the euclidean space $\mathbb{R}^d$, equipped with a suitable topology.

In this talk I will explain Goodwillie's method for understanding the homotopy type of that space of embeddings, which generalize Smale's approach for understanding spaces of immersions.

I will then show how Kontsevich's formality of the little cube operad can be used to understand better the rational homology of these spaces of embeddings. In particular we get an explicit description of the rational homology of the space of embeddings of the circle in high-dimensional euclidean spaces, i.e. of high-codimensional knots. This is a joint work with Greg Arone, Victor Turchin, and Ismar Volić.

Andrey Lazarev (Leicester) *Characteristic classes of operadic algebras, Morita theory and cyclic cohomology*

Abstract: Joint with J. Chuang. Around 15 years ago Kontsevich gave an amazing construction of a family of cohomology classes on the uncompactified moduli space of Riemann surfaces from an A-infinity algebra with a kind of Poincare duality. In the interim period the construction has been vastly generalized and understood from two conceptually different points of view. One is based on the homotopy

theory of (modular) operads, the other – on Gelfand-Fuks type cohomology of certain infinite-dimensional Lie algebras. I will describe this construction and discuss new results and open questions.

Muriel Livernet (Paris 13) *E_2-homology for commutative algebras via functor homology.*

ABSTRACT: There are many homology theories that can be applied to commutative algebras; in this talk we are concerned with homology theories associated via operad theory to e_n-algebras. For instance, an e_1 algebra is an associative algebra, an e_2 algebra is an associative algebra where the product is commutative up to a 1- homotopy (controlled by the $\cup_1$ product), an e_3-algebra has its associative product commutative up to a 2-homotopy (the cup_1 product is commutative up to a 1-homotopy controlled by the cup_2-product) and so on.... An e_∞-algebra is commutative up to all homotopies. Hence, a commutative algebra is an e_n-algebra for any n, and one can apply the homology theory associated to e_n-algebras to a commutative algebra. For e_1-algebras the homology considered is the Hochschild homology, for e_∞-algebras the homology considered is the Gamma homology of Robinson and Whitehouse. These two homology theories have an interpretation as Tor functor in suitable categories of functors (proved by Pirashvili and Richter). We prove that this result holds also for the homology theory associated to e_2 algebras. (Joint work with Birgit Richter)

Sergei Merkulov (Stockholm) *De Rham Field theories on configuration spaces and the Grothendieck-Teichmuller group.*

ABSTRACT: We discuss a de Rham Field theory on a compactified (braid) configuration space of pairwise distinct points in the hyperbolic n-space, and its applications to exotic automorphisms of some geometric and algebraic structures such as Schouten's prop of polyvector Fields ($n = 2$) and Drinfeld's prop of quasi Lie bialgebras ($n = 3$). The theory depends on the choice of a Kontsevich-type propagator, a certain differential $(n-1)$-form on the n-dimensional hyperbolic space.

Joan Millès (Nice) *André-Quillen cohomology theory of an algebra over an operad.*

ABSTRACT: Following the ideas of Quillen and by means of model category structures, Hinich, Goerss and Hopkins have developed a cohomology theory for (simplicial) algebras over a (simplicial) operad. Thank to Koszul duality theory of operads, we make these theories explicit in the differential graded setting.

We recover the known theories as Hochschild cohomology theory for associative algebras and Chevalley-Eilenberg cohomology theory for Lie algebras and we define the new case of homotopy algebras.

We study the general properties of such cohomology theories and we give an effective criterium to determine whether a cohomology theory is an Ext-functor. We show that it is always the case for homotopy algebras.

Ieke Moerdijk (Utrecht) *Dendroidal sets and topological operads.*
ABSTRACT: The category of dendroidal sets is an extension of that of simplicial sets. It carries a Quillen model structure, whose fibrant objects are the infinity operads, and whose associated homotopy theory is equivalent to the homotopy theory of topological operads. This is joint work with Cisinski, and extends some results for simplicial sets and topological categories proved by Bergner, Joyal and Lurie.

Paolo Salvatore (Roma) *Cyclic formality of the framed little discs.*
ABSTRACT: The framed little 2-discs operad is equivalent to the cyclic operad of bordered Riemann spheres. We show that this operad is formal as a cyclic operad. This is joint work with Giansiracusa.

Travis Schedler (MIT) *Twisted algebras, pre-Lie algebras, and wheelgebras.*
ABSTRACT: For any operad O, it is interesting to consider not just usual algebras over O, but twisted algebras, i.e., algebras in the category of symmetric sequences of vector spaces, rather than vector spaces. This replaces free commutative algebras, $Sym(V)$, with free associative ones, $T(V)$, and Lie structures on V, i.e., Poisson structures of degree -1, with pre-Lie structures on V. The construction applies to Connes and Kreimer's renormalization Hopf algebras and the PBW theorem for pre-Lie algebras, and the classical Yang-Baxter equation arises surprisingly. Similarly, one may consider wheelgebras rather than twisted algebras (algebras in the category of wheelspaces), which recovers wheeled PROPs and noncommutative differential geometry on associative algebras.

Dev Sinha *Hopf invariants and configuration spaces.*
ABSTRACT: We start by giving a new construction, based on the Lie cooperad, of a complete set of rational homotopy functionals for simply connected spaces (after Sullivan, Haefliger, Hain and Novikov). This construction has both good formal properties, being based on a Quillen functor, and is geometrically rich, showing that rational homotopy functionals are all given by generalized linking numbers. The

key combinatorics arises from the geometry of ordered configuration spaces. We then share the beginnings of our attempt to make similar constructions in characteristic p. Namely, we have found a new product in the cohomology of unordered configuration spaces and shown that Stiefel-Whitney classes generate the cohomology of symmetric groups under these two products. The work on Hopf invariants is joint with Ben Walter, and the work on symmetric groups is joint with Paolo Salvatore.

Henrik Strohmayer (Stockholm) *Prop profiles of compatible Poisson and Nijenhuis structures.*

ABSTRACT: Recently S.A. Merkulov established a link between differential geometry and homological algebra by giving descriptions of several differential geometric structures in terms of minimal resolutions of props, so called prop profiles. In this talk we will show how this correspondence can be interpreted in terms of isomorphisms of Lie algebras; representations of minimal resolutions of props and e.g. Poisson and Nijenhuis structures can all be interpreted as Maurer-Cartan elements in certain Lie algebras. In order to compute minimal resolutions of the involved props, Koszulness is essential. We also present the prop profiles of compatible Poisson and Nijenhuis structures and give a geometrical interpretation of the former in terms of a family of brackets on the structure sheaf.

Mark Weber (Paris 7) *Operads and tensor products in higher dimensional category theory.*

ABSTRACT: In this talk a correspondence between the globular operads used in higher category theory, and lax tensor products will be given. Under this correspondence the algebras of the given globular operad coincide with categories enriched in the corresponding lax monoidal structure. An application of this theory is that one can capture the Gray tensor product of 2-categories formally from the operad for Gray categories, and the possibility exists to extend this example to shed some light on the question of what higher dimensional semi-strict categories may be in general. (Joint work with Michael Batanin and Denis-Charles Cisinski)