Exposés de Marc Levine au LAGA, septembre 2025

HORAIRES: DE 10H30 À 11H30 LES 23, 29 ET 30 SEPTEMBRE 2025

LIEU: SALLE B407, LAGA, CAMPUS DE VILLETANEUSE

1. *Six functors, $\mathbf{A}^1$ Euler characteristic and quadratic Euler classes*

   I will give a brief description of the six-functor formalism for the motivic stable homotopy category, and how this leads to the pair cohomology/Borel Moore homology for an arbitrary motivic commutative ring spectrum. Morel's theorem gives the categorical Euler characteristic of a dualizable object in $SH(k)$ as an element (the "$\mathbf{A}^1$ Euler characteristic") in the Grothendieck–Witt ring $GW(k)$. We introduce the motivic Euler class of a vector bundle, describe the motivic Gauss–Bonnet theorem of Déglise–Jin–Khan and show how one can use this to compute the $\mathbf{A}^1$ Euler characteristic of smooth projective varieties in a number of ways.

2. *A calculus of Euler classes in Witt–sheaf cohomology*

   We look at the special case of the motivic Eilenberg–MacLane spectrum associated to the sheaf of Witt rings, and develop a calculus of characteristic classes parallel to the classical theory of Chern classes in the Chow ring. We introduce localization methods and show how these are used to compute quadratic versions of many classical enumerative problems: lines and twisted cubics on hypersufaces,, as well as some dimension 0 Donaldson–Thomas invariants.

3. *Quadratic curve counting*

   We describe our work with Kass, Solomon and Wickelgren on defining quadratic curve counts for rational curves on del Pezzo surfaces. We review computational methods: tropical methods of Jaramillo Puentes–Markwig–Pauli–Röhrle, extended by wall-crossing formulas due to Jaramillo Puentes, Brugallé–Wickelgren, as well as a general conjecture for the quadratic curve counts, verified for k-rational del Pezzo surfaces of degree at least 6, by Brugallé–Rau–Wickelgren.